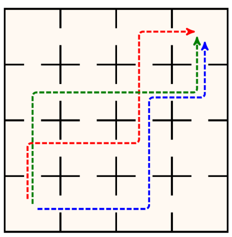
University of Washington Math Hour Olympiad, 2019

Grades 8–10

1. The alphabet of the Aau-Bau language consists of two letters: A and B. Two words have the same meaning if one of them can be constructed from the other by replacing any AA with A, replacing any BB with B, or by replacing any ABA with BAB. For example, the word AABA means the same thing as ABA, and AABA also means the same thing as ABAB.

In this language, is it possible to name all seven days of the week?

2. A museum has a 4×4 grid of rooms. Every two rooms that share a wall are connected by a door. Each room contains some paintings.

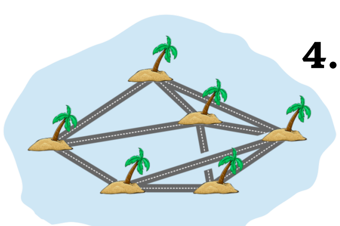


The total number of paintings along any path of 7 rooms from the lower left to the upper right room is always the same. Furthermore, the total number of paintings along any path of 7 rooms from the lower right to the upper left room is always the same.

The guide states that the museum has exactly 500 paintings. Show that the guide is mistaken.

3. A playground has a swing-set with exactly three swings. When 3^{rd} and 4^{th} graders from Dr. Anna's math class play during recess, she has a rule that if a 3^{rd} grader is in the middle swing there must be 4^{th} graders on that person's left and right. And if there is a 4^{th} grader in the middle, there must be 3^{rd} graders on that person's left and right.

Dr. Anna calculates that there are 350 different ways her students can arrange themselves on the three swings with no empty seats. How many students are in her class?



4. The archipelago Artinagos has 19 islands. Each island has toll bridges to at least 3 other islands. An unsuspecting driver used a bad mapping app to plan a route from North Noether Island to South Noether Island, which involved crossing 12 bridges. Show that there must be a route with fewer bridges.

5. Is it possible to place the numbers from 1 to 121 in an 11×11 table so that numbers that differ by 1 are in horizontally or vertically adjacent cells and all the perfect squares $(1, 4, 9, \ldots, 121)$ are in one column?