

University of Washington Math Hour Olympiad, 2016

Grades 8-10

1. Alice and Bob compiled a list of movies that exactly one of them saw, then Cindy and Dale did the same. To their surprise, these two lists were identical. Prove that if Alice and Cindy list all movies that exactly one of them saw, this list will be identical to the one for Bob and Dale.



2. Several whole rounds of cheese were stored in a pantry. One night some rats sneaked in and consumed 10 of the rounds, each rat eating an equal portion. Some were satisfied, but 7 greedy rats returned the next night to finish the remaining rounds. Their portions on the second night happened to be half as large as on the first night. How many rounds of cheese were initially in the pantry?



3. You have 100 pancakes, one with a single blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are stacked in a random order. Count the number of blueberries in the top pancake, and call that number N . Pick up the stack of the top N pancakes, and flip it upside down. Prove that if you repeat this counting-and-flipping process, the pancake with one blueberry will eventually end up at the top of the stack.

4. There are two lemonade stands along the 4-mile-long circular road that surrounds Sour Lake. 100 children live in houses along the road. Every day, each child buys a glass of lemonade from the stand that is closest to her house, as long as she does not have to walk more than one mile along the road to get there.



A stand's *advantage* is the difference between the number of glasses it sells and the number of glasses its competitor sells. The stands are positioned such that neither stand can increase its advantage by moving to a new location, if the other stand stays still. What is the maximum number of kids who can't buy lemonade (because both stands are too far away)?



5. Merlin uses several spells to move around his 64-room castle. When Merlin casts a spell in a room, he ends up in a different room of the castle. Where he ends up only depends on the room where he cast the spell and which spell he cast. The castle has the following magic property: if a sequence of spells brings Merlin from some room A back to room A , then from any other room B in the castle, that same sequence brings Merlin back to room B .

Prove that there are two different rooms X and Y and a sequence of spells that both takes Merlin from X to Y and from Y to X .