

University of Washington Math Hour Olympiad, 2016

Grades 6-7

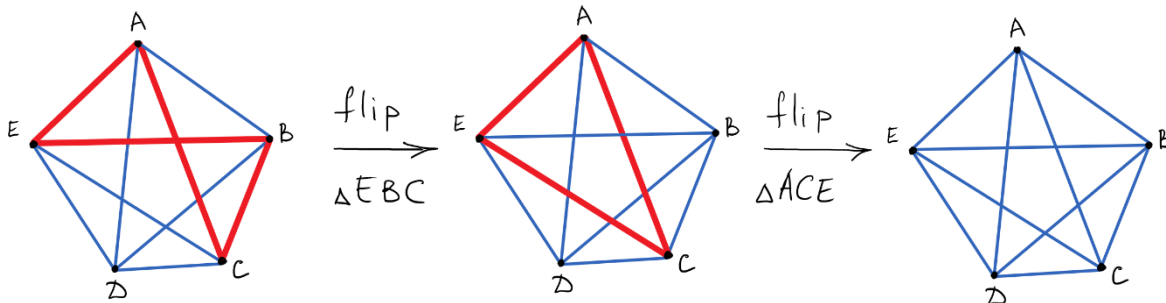
1. At a fortune-telling exam, 13 witches are sitting in a circle. To pass the exam, a witch must correctly predict, for everybody except herself and her two neighbors, whether they will pass or fail. Each witch predicts that each of the 10 witches she is asked about will fail. How many witches could pass?

2. Out of 152 coins, 7 are counterfeit. All counterfeit coins have the same weight, and all real coins have the same weight, but counterfeit coins are lighter than real coins. How can you find 19 real coins if you are allowed to use a balance scale three times?



3. The digits of a number N increase from left to right. What could the sum of the digits of $9 \times N$ be?

4. The sides and diagonals of a pentagon are colored either blue or red. You can choose three vertices and flip the colors of all three lines that join them. Can every possible coloring be turned all blue by a sequence of such moves?



5. You have 100 pancakes, one with a single blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are stacked in a random order. Count the number of blueberries in the top pancake and call that number N . Pick up the stack of the top N pancakes and flip it upside down. Prove that if you repeat this counting-and-flipping process, the pancake with one blueberry will eventually end up at the top of the stack.