

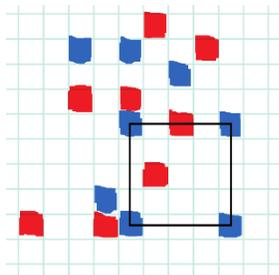
# University of Washington Math Hour Olympiad, 2015

## Grades 8-10

1. Six pirates – Captain Jack and his five crewmen – sit in a circle to split a treasure of 99 gold coins. Jack must decide how many coins to take for himself and how many to give each crewman (not necessarily the same number to each). The five crewmen will then vote on Jack's decision. Each is greedy and will vote “aye” only if he gets more coins than each of his two neighbors. If a majority vote “aye”, Jack's decision is accepted. Otherwise Jack is thrown overboard and gets nothing.



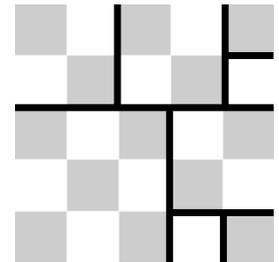
What is the most coins Captain Jack can take for himself and survive?



2. Rose and Bella take turns painting cells red and blue on an infinite piece of graph paper. On Rose's turn, she picks any blank cell and paints it red. Bella, on her turn, picks any blank cell and paints it blue. Bella wins if the paper has four blue cells arranged as corners of a square of any size with sides parallel to the grid lines. Rose goes first. Show that she cannot prevent Bella from winning.

3. A  $25 \times 25$  checkerboard is cut along the gridlines into some number of smaller square boards. Show that the total length of the cuts is divisible by 4.

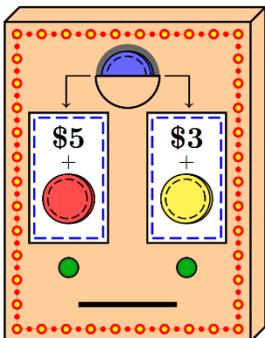
For example, the cuts shown on the picture have total length 16, which is divisible by 4.



4. Each robot in the Martian Army is equipped with a battery that lasts some number of hours.

For any two robots, one's battery lasts at least three times as long as the other's. A robot works until its battery is depleted, then recharges its battery until it is full, then goes back to work, and so on. A battery that lasts  $N$  hours takes exactly  $N$  hours to recharge.

Prove that there will be a moment in time when all the robots are recharging (so you can invade the planet).



5. A casino machine accepts tokens of 32 different colors, one at a time. For each color, the player can choose between two fixed rewards. Each reward is up to \$10 cash, plus maybe another token. For example, a blue token always gives the player a choice of getting either \$5 plus a red token or \$3 plus a yellow token; a black token can always be exchanged either for \$10 (but no token) or for a brown token (but no cash). A player may keep playing as long as he has a token.

Rob and Bob each have one white token. Rob watches Bob play and win \$500. Prove that Rob can win at least \$1000.