

University of Washington Math Hour Open Olympiad, 2012

Grades 8-10

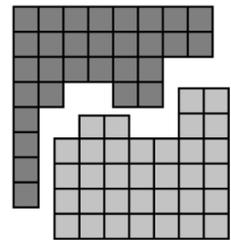
1. In the Hundred Acre Wood, all the animals are either knights or liars. Knights always tell the truth and liars always lie. One day in the Wood, Winnie-the-Pooh, a knight, decides to visit his friend Rabbit, also a noble knight. Upon arrival, Pooh finds his friend sitting at a round table with 5 other guests.

One-by-one, Pooh asks each person at the table how many of his two neighbors are knights. Surprisingly, he gets the same answer from everybody! "Oh bother!" proclaims Pooh. "I still don't have enough information to figure out how many knights are at this table."

"But it's my birthday," adds one of the guests. "Yes, it's his birthday!" agrees his neighbor.

Now Pooh can tell how many knights are at the table. Can you?

2. Harry has an 8×8 board filled with the numbers 1 and -1, and the sum of all 64 numbers is 0. A magical cut of this board is a way of cutting it into two pieces so that the sum of the numbers in each piece is also 0. The pieces should not have any holes. Prove that Harry will always be able to find a magical cut of his board. (The picture shows an example of a proper cut.)



3. Several girls participate in a tennis tournament in which each player plays each other player exactly once. At the end of the tournament, it turns out that each player has lost at least one of her games. Prove that it is possible to find three players A, B, and C such that A defeated B, B defeated C, and C defeated A.
4. 120 bands are participating in this year's Northwest Grunge Rock Festival, and they have 119 fans in total. Each fan belongs to exactly one fan club. A fan club is called crowded if it has at least 15 members.



Every morning, all the members of one of the crowded fan clubs start arguing over who loves their favorite band the most. As a result of the fighting, each of them leaves the club to join another club, but no two of them join the same one.

Is it true that, no matter how the clubs are originally arranged, all these arguments will eventually stop?

5. In Infinite City, the streets form a grid of squares extending infinitely in all directions. Bonnie and Clyde have just robbed the Infinite City Bank, located at the busiest intersection downtown. Bonnie sets off heading north on her bike, and, 30 seconds later, Clyde bikes after her in the same direction. They each bike at a constant speed of 1 block per minute. In order to throw off any authorities, each of them must turn either left or right at every intersection. If they continue biking in this manner, will they ever be able to meet?

