

University of Washington Math Hour Open Olympiad, 2012

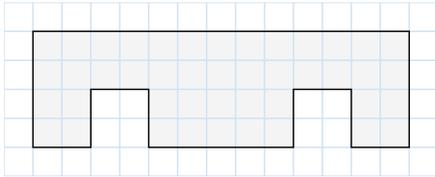
Grades 5-7

1. Tom and Jerry stole a chain of 7 sausages and are now trying to divide the bounty. They take turns biting the sausages at one of the connections. When one of them breaks a connection, he may eat any single sausages that may fall out. Tom takes the first bite. Each of them is trying his best to eat more sausages than his opponent. Who will succeed?

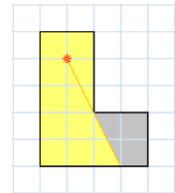


2. The King of the Mountain Dwarves wants to light his underground throne room by placing several torches so that the whole room is lit. The king, being very miserly, wants to use as few torches as possible. What is the least number of torches he could use? (You should show why he can't do it with a smaller number of torches.)

This is the shape of the throne room:



Also, the walls in all rooms are lined with velvet and do not reflect the light. For example, the picture on the right shows how another room in the castle is partially lit.



3. In the Hundred Acre Wood, all the animals are either knights or liars. Knights always tell the truth and liars always lie. One day in the Wood, Winnie-the-Pooh, a knight, decides to visit his friend Rabbit, also a noble knight. Upon arrival, Pooh finds his friend sitting at a round table with 5 other guests.

One-by-one, Pooh asks each person at the table how many of his two neighbors are knights. Surprisingly, he gets the same answer from everybody! "Oh bother!" proclaims Pooh. "I still don't have enough information to figure out how many knights are at this table."

"But it's my birthday," adds one of the guests. "Yes, it's his birthday!" agrees his neighbor.

Now Pooh can tell how many knights are at the table. Can you?



4. Several girls participate in a tennis tournament in which each player plays each other player exactly once. At the end of the tournament, it turns out that each player has lost at least one of her games. Prove that it is possible to find three players A, B, and C such that A defeated B, B defeated C, and C defeated A.

5. There are 40 piles of stones with an equal number of stones in each. Two players, Ann and Bob, can select any two piles of stones and combine them into one bigger pile, as long as this pile would not contain more than half of all the stones on the table. A player who can't make a move loses. Ann goes first. Who wins?

