

University of Washington Math Hour Open Olympiad, 2011

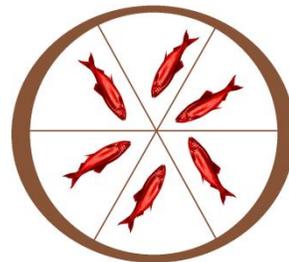
Grades 6-7

1. In a chemical lab there are three vials: one that can hold 1 oz of fluid, another that can hold 2 oz, and a third that can hold 3 oz. The first is filled with grape juice, the second with sulfuric acid, and the third with water. There are also 3 empty vials in the cupboard, also of sizes 1 oz, 2 oz, and 3 oz. In order to save the world with grape-flavored acid, James Bond must make three full bottles, one of each size, filled with a mixture of all three liquids so that each bottle has the same ratio of juice to acid to water. How can he do this, considering he was silly enough not to bring any equipment?



2. Twelve people, some are knights and some are knaves, are sitting around a table. Knaves always lie and knights always tell the truth. At some point they start up a conversation. The first person says, "There are no knights around this table." The second says, "There is at most one knight at this table." The third - "There are at most two knights at the table." And so on until the 12th says, "There are at most eleven knights at the table." How many knights are at the table? Justify your answer.

3. Aquaman has a barrel divided up into six sections, and he has placed a red herring in each. Aquaman can command any fish of his choice to either 'jump counterclockwise to the next sector' or 'jump clockwise to the next sector.' Using a sequence of exactly 30 of these commands, can he relocate all the red herrings to one sector? If yes, show how. If no, explain why not.



4. Is it possible to place 13 integers around a circle so that the sum of any 3 adjacent numbers is exactly 13?
5. Two girls are playing a game. The first player writes the letters A or B in a row, left to right, adding one letter on her turn. The second player switches any two letters after each move by the first player (the letters do not have to be adjacent), or does nothing, which also counts as a move. The game is over when each player has made 2011 moves. Can the second player plan her moves so that the resulting letters form a palindrome? (A palindrome is a sequence that reads the same forward and backwards, e.g. AABABAA.)

AABABAA